Edexcel Maths FP2

Past Paper Pack

2009-2013

Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						6 6 6 8 / 0 1	Signature	

Paper Reference(s)

6668/01

Edexcel GCE

Further Pure Mathematics FP2

Advanced/Advanced Subsidiary

Friday 19 June 2009 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Orange)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
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Instructions to Candidates

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Check that you have the correct question paper.
Answer ALL the questions. You must write your answer to each question in the space following the question.
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Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this question paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

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1. (a) Express $\dfrac{1}{r(r+2)}$ in partial fractions.

(1)

(b) Hence show that $\displaystyle\sum_{r=1}^{n} \frac{4}{r(r+2)} = \frac{n(3n+5)}{(n+1)(n+2)}$.

(5)

Leave blank

2. Solve the equation

$$z^3 = 4\sqrt{2} - 4\sqrt{2}\mathrm{i},$$

giving your answers in the form $r(\cos\theta + \mathrm{i}\sin\theta)$, where $-\pi < \theta \leqslant \pi$.

(6)

Leave blank

3. Find the general solution of the differential equation

$$\sin x \frac{\mathrm{d}y}{\mathrm{d}x} - y\cos x = \sin 2x \sin x,$$

giving your answer in the form $y = \mathrm{f}(x)$.

(8)

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4.

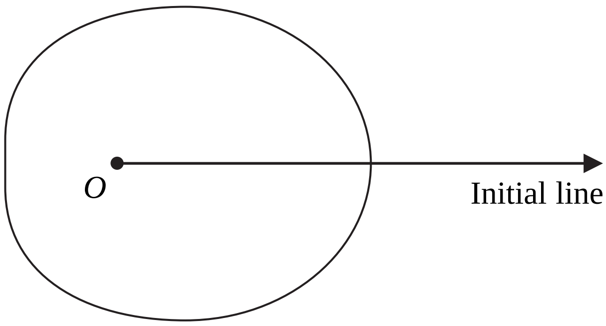

Figure 1

Figure 1 shows a sketch of the curve with polar equation

$$r = a + 3\cos\theta, \quad a > 0, \quad 0 \leqslant \theta < 2\pi$$

The area enclosed by the curve is $\frac{107}{2}\pi$.

Find the value of a.

(8)

Leave blank

Question 4 continued

Leave blank

5.

$$y = \sec^2 x$$

(a) Show that $\dfrac{d^2y}{dx^2} = 6\sec^4 x - 4\sec^2 x$.

(4)

(b) Find a Taylor series expansion of $\sec^2 x$ in ascending powers of $\left(x - \dfrac{\pi}{4}\right)$, up to and including the term in $\left(x - \dfrac{\pi}{4}\right)^3$.

(6)

Leave blank

Question 5 continued

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6. A transformation T from the z-plane to the w-plane is given by

$$w = \frac{z}{z + \mathrm{i}}, \quad z \neq -\mathrm{i}$$

The circle with equation $|z| = 3$ is mapped by T onto the curve C.

(a) Show that C is a circle and find its centre and radius.

(8)

The region $|z| < 3$ in the z-plane is mapped by T onto the region R in the w-plane.

(b) Shade the region R on an Argand diagram.

(2)

Leave blank

Question 6 continued

Leave blank

7. (a) Sketch the graph of $y = |x^2 - a^2|$, where $a > 1$, showing the coordinates of the points where the graph meets the axes.

(2)

(b) Solve $|x^2 - a^2| = a^2 - x$, $a > 1$.

(6)

(c) Find the set of values of x for which $|x^2 - a^2| > a^2 - x$, $a > 1$.

(4)

Leave blank

Question 7 continued

Leave blank

8.

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 2e^{-t}$$

Given that $x = 0$ and $\frac{dx}{dt} = 2$ at $t = 0$,

(a) find x in terms of t.

(8)

The solution to part (a) is used to represent the motion of a particle P on the x-axis. At time t seconds, where $t > 0$, P is x metres from the origin O.

(b) Show that the maximum distance between O and P is $\frac{2\sqrt{3}}{9}$ m and justify that this distance is a maximum.

(7)

Leave blank

Question 8 continued

Q8

(Total 15 marks)

TOTAL FOR PAPER: 75 MARKS

END

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Paper Reference(s)

6668/01

Edexcel GCE

Further Pure Mathematics FP2

Advanced/Advanced Subsidiary

Thursday 24 June 2010 – Morning

Time: 1 hour 30 minutes

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Team Leader's use only

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1. (a) Express $\dfrac{3}{(3r-1)(3r+2)}$ in partial fractions.

(2)

(b) Using your answer to part (a) and the method of differences, show that

$$\sum_{r=1}^{n} \frac{3}{(3r-1)(3r+2)} = \frac{3n}{2(3n+2)}$$

(3)

(c) Evaluate $\displaystyle\sum_{r=100}^{1000} \frac{3}{(3r-1)(3r+2)}$, giving your answer to 3 significant figures.

(2)

Leave blank

2. The displacement x metres of a particle at time t seconds is given by the differential equation

$$\frac{d^2x}{dt^2} + x + \cos x = 0$$

When $t = 0$, $x = 0$ and $\frac{dx}{dt} = \frac{1}{2}$.

Find a Taylor series solution for x in ascending powers of t, up to and including the term in t^3.

(5)

Leave blank

3. (a) Find the set of values of x for which

$$x+4>\frac{2}{x+3}$$

(6)

(b) Deduce, or otherwise find, the values of x for which

$$x+4>\frac{2}{|x+3|}$$

(1)

Leave blank

4. $$z = -8 + (8\sqrt{3})\text{i}$$

(a) Find the modulus of z and the argument of z.

(3)

Using de Moivre's theorem,

(b) find z^3,

(2)

(c) find the values of w such that $w^4 = z$, giving your answers in the form $a + \text{i}b$, where $a, b \in \mathbb{R}$.

(5)

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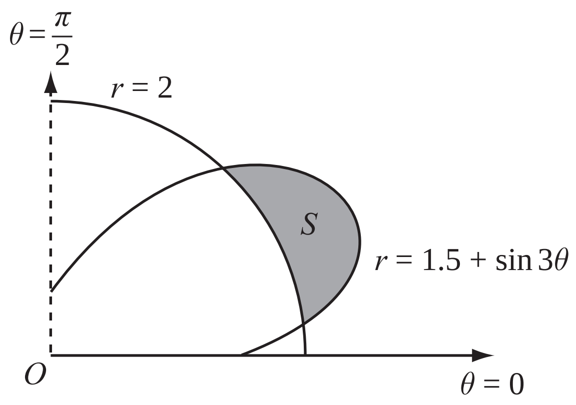

Figure 1

Figure 1 shows the curves given by the polar equations

$$r = 2, \qquad 0 \leqslant \theta \leqslant \frac{\pi}{2},$$

$$\text{and} \qquad r = 1.5 + \sin 3\theta, \qquad 0 \leqslant \theta \leqslant \frac{\pi}{2}.$$

(a) Find the coordinates of the points where the curves intersect.

(3)

The region S, between the curves, for which $r > 2$ and for which $r < (1.5 + \sin 3\theta)$, is shown shaded in Figure 1.

(b) Find, by integration, the area of the shaded region S, giving your answer in the form $a\pi + b\sqrt{3}$, where a and b are simplified fractions.

(7)

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Question 5 continued

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6. A complex number z is represented by the point P in the Argand diagram.

(a) Given that $|z-6| = |z|$, sketch the locus of P.

(2)

(b) Find the complex numbers z which satisfy both $|z-6| = |z|$ and $|z-3-4\mathrm{i}| = 5$.

(3)

The transformation T from the z-plane to the w-plane is given by $w = \dfrac{30}{z}$.

(c) Show that T maps $|z-6| = |z|$ onto a circle in the w-plane and give the cartesian equation of this circle.

(5)

Leave blank

Question 6 continued

Leave blank

7. (a) Show that the transformation $z = y^{\frac{1}{2}}$ transforms the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} - 4y\tan x = 2y^{\frac{1}{2}} \qquad \text{(I)}$$

into the differential equation

$$\frac{\mathrm{d}z}{\mathrm{d}x} - 2z\tan x = 1 \qquad \text{(II)}$$

(5)

(b) Solve the differential equation (II) to find z as a function of x.

(6)

(c) Hence obtain the general solution of the differential equation (I).

(1)

Leave blank

Question 7 continued

Leave blank

8. (a) Find the value of λ for which $y = \lambda x \sin 5x$ is a particular integral of the differential equation

$$\frac{d^2y}{dx^2} + 25y = 3\cos 5x$$

(4)

(b) Using your answer to part (a), find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 25y = 3\cos 5x$$

(3)

Given that at $x = 0$, $y = 0$ and $\frac{dy}{dx} = 5$,

(c) find the particular solution of this differential equation, giving your solution in the form $y = f(x)$.

(5)

(d) Sketch the curve with equation $y = f(x)$ for $0 \leqslant x \leqslant \pi$.

(2)

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Question 8 continued

Q8

(Total 14 marks)

TOTAL FOR PAPER: 75 MARKS

END

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Paper Reference(s)

6668/01

Edexcel GCE

Further Pure Mathematics FP2

Advanced/Advanced Subsidiary

Thursday 23 June 2011 – Morning

Time: 1 hour 30 minutes

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Question Number	Leave Blank
1	
2	
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1. Find the set of values of x for which

$$\frac{3}{x+3} > \frac{x-4}{x}$$

(7)

Leave blank

2.

$$\frac{d^2y}{dx^2} = e^x\left(2y\frac{dy}{dx} + y^2 + 1\right)$$

(a) Show that

$$\frac{d^3y}{dx^3} = e^x\left[2y\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 + ky\frac{dy}{dx} + y^2 + 1\right],$$

where k is a constant to be found.

(3)

Given that, at $x = 0$, $y = 1$ and $\frac{dy}{dx} = 2$,

(b) find a series solution for y in ascending powers of x, up to and including the term in x^3.

(4)

P 3 5 4 1 3 A 0 4 2 8

Leave blank

3. Find the general solution of the differential equation

$$x\frac{\mathrm{d}y}{\mathrm{d}x}+5y=\frac{\ln x}{x}, \qquad x>0$$

giving your answer in the form $y=\mathrm{f}(x)$.

(8)

Leave blank

4. Given that

$$(2r+1)^3 = Ar^3 + Br^2 + Cr + 1,$$

(a) find the values of the constants A, B and C.

(2)

(b) Show that

$$(2r+1)^3 - (2r-1)^3 = 24r^2 + 2$$

(2)

(c) Using the result in part (b) and the method of differences, show that

$$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$$

(5)

Leave blank

Question 4 continued

Leave blank

5. The point P represents the complex number z on an Argand diagram, where

$$|z - \mathrm{i}| = 2$$

The locus of P as z varies is the curve C.

(a) Find a cartesian equation of C.

(2)

(b) Sketch the curve C.

(2)

A transformation T from the z-plane to the w-plane is given by

$$w = \frac{z + \mathrm{i}}{3 + \mathrm{i}z}, \quad z \neq 3\mathrm{i}$$

The point Q is mapped by T onto the point R. Given that R lies on the real axis,

(c) show that Q lies on C.

(5)

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Question 5 continued

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6.

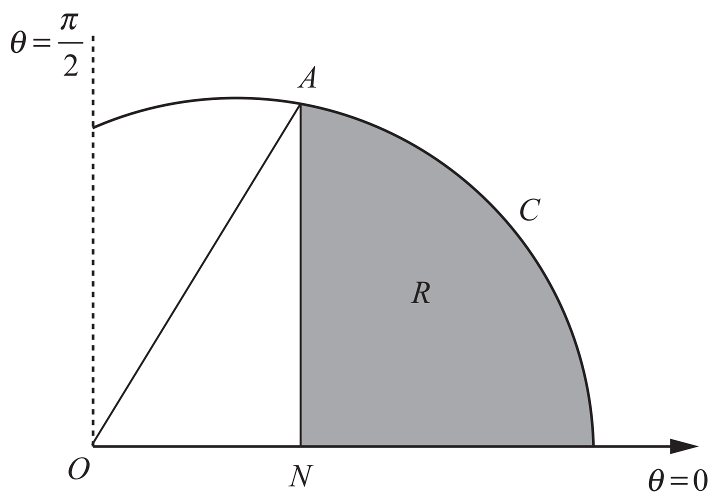

Figure 1

The curve C shown in Figure 1 has polar equation

$$r = 2 + \cos\theta, \quad 0 \leqslant \theta \leqslant \frac{\pi}{2}$$

At the point A on C, the value of r is $\frac{5}{2}$.

The point N lies on the initial line and AN is perpendicular to the initial line.

The finite region R, shown shaded in Figure 1, is bounded by the curve C, the initial line and the line AN.

Find the exact area of the shaded region R.

(9)

Leave blank

Question 6 continued

Leave blank

7. (a) Use de Moivre's theorem to show that

$$\sin 5\theta = 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta$$

(5)

Hence, given also that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$,

(b) find all the solutions of

$$\sin 5\theta = 5\sin 3\theta,$$

in the interval $0 \leqslant \theta < 2\pi$. Give your answers to 3 decimal places.

(6)

Leave blank

Question 7 continued

Leave blank

8. The differential equation

$$\frac{d^2x}{dt^2}+6\frac{dx}{dt}+9x=\cos 3t, \quad t\geqslant 0$$

describes the motion of a particle along the x-axis.

(a) Find the general solution of this differential equation.
(8)

(b) Find the particular solution of this differential equation for which, at $t=0$,

$x=\frac{1}{2}$ and $\frac{dx}{dt}=0$.
(5)

On the graph of the particular solution defined in part (b), the first turning point for $t>30$ is the point A.

(c) Find approximate values for the coordinates of A.
(2)

Leave blank

Question 8 continued

Q8

(Total 15 marks)

TOTAL FOR PAPER: 75 MARKS

END

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Paper Reference(s)

6668/01

Edexcel GCE

Further Pure Mathematics FP2

Advanced/Advanced Subsidiary

Friday 22 June 2012 – Afternoon

Time: 1 hour 30 minutes

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Question Number	Leave Blank
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Turn over

PEARSON

Leave blank

1. Find the set of values of x for which

$$|x^2 - 4| > 3x$$

(5)

Leave blank

2. The curve C has polar equation

$$r = 1 + 2\cos\theta, \quad 0 \leqslant \theta \leqslant \frac{\pi}{2}$$

At the point P on C, the tangent to C is parallel to the initial line.

Given that O is the pole, find the exact length of the line OP.

(7)

Leave blank

3. (a) Express the complex number $-2+\left(2\sqrt{3}\right)\mathrm{i}$ in the form $r(\cos\theta+\mathrm{i}\sin\theta)$, $-\pi<\theta\leqslant\pi$.
(3)

(b) Solve the equation

$$z^4=-2+\left(2\sqrt{3}\right)\mathrm{i}$$

giving the roots in the form $r(\cos\theta+\mathrm{i}\sin\theta)$, $-\pi<\theta\leqslant\pi$.
(5)

Leave blank

4. Find the general solution of the differential equation

$$\frac{d^2x}{dt^2}+5\frac{dx}{dt}+6x=2\cos t-\sin t$$

(9)

Leave blank

Question 4 continued

Leave blank

5. $$x\frac{dy}{dx} = 3x + y^2$$

(a) Show that

$$x\frac{d^2y}{dx^2} + (1-2y)\frac{dy}{dx} = 3$$

(2)

Given that $y = 1$ at $x = 1$,

(b) find a series solution for y in ascending powers of $(x-1)$, up to and including the term in $(x-1)^3$.

(8)

Leave blank

Question 5 continued

Leave blank

6. (a) Express $\dfrac{1}{r(r+2)}$ in partial fractions.

(2)

(b) Hence prove, by the method of differences, that

$$\sum_{r=1}^{n} \frac{1}{r(r+2)} = \frac{n(an+b)}{4(n+1)(n+2)}$$

where a and b are constants to be found.

(6)

(c) Hence show that

$$\sum_{r=n+1}^{2n} \frac{1}{r(r+2)} = \frac{n(4n+5)}{4(n+1)(n+2)(2n+1)}$$

(3)

Leave blank

Question 6 continued

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7. (a) Show that the substitution $y = vx$ transforms the differential equation

$$3xy^2 \frac{dy}{dx} = x^3 + y^3 \qquad \text{(I)}$$

into the differential equation

$$3v^2 x \frac{dv}{dx} = 1 - 2v^3 \qquad \text{(II)}$$

(3)

(b) By solving differential equation (II), find a general solution of differential equation (I) in the form $y = f(x)$.

(6)

Given that $y = 2$ at $x = 1$,

(c) find the value of $\frac{dy}{dx}$ at $x = 1$

(2)

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Question 7 continued

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8. The point P represents a complex number z on an Argand diagram such that

$$|z - 6\mathrm{i}| = 2|z - 3|$$

(a) Show that, as z varies, the locus of P is a circle, stating the radius and the coordinates of the centre of this circle.

(6)

The point Q represents a complex number z on an Argand diagram such that

$$\arg(z - 6) = -\frac{3\pi}{4}$$

(b) Sketch, on the same Argand diagram, the locus of P and the locus of Q as z varies.

(4)

(c) Find the complex number for which both $|z - 6\mathrm{i}| = 2|z - 3|$ and $\arg(z - 6) = -\dfrac{3\pi}{4}$

(4)

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Question 8 continued

Q8

(Total 14 marks)

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Friday 21 June 2013 – Morning

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PEARSON

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1. A transformation T from the z-plane to the w-plane is given by

$$w = \frac{z + 2\mathrm{i}}{\mathrm{i}z} \qquad z \neq 0$$

The transformation maps points on the real axis in the z-plane onto a line in the w-plane.

Find an equation of this line.

(4)

Leave blank

2. Use algebra to find the set of values of x for which

$$\frac{6x}{3-x} > \frac{1}{x+1}$$

(7)

Leave blank

3. (a) Express $\dfrac{2}{(r+1)(r+3)}$ in partial fractions.

(2)

(b) Hence show that

$$\sum_{r=1}^{n} \frac{2}{(r+1)(r+3)} = \frac{n(5n+13)}{6(n+2)(n+3)}$$

(4)

(c) Evaluate $\displaystyle\sum_{r=10}^{100} \frac{2}{(r+1)(r+3)}$, giving your answer to 3 significant figures.

(2)

Leave blank

Question 3 continued

Leave blank

4. Given that

$$y\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 + 5y = 0$$

(a) find $\frac{\mathrm{d}^3 y}{\mathrm{d}x^3}$ in terms of $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}$, $\frac{\mathrm{d}y}{\mathrm{d}x}$ and y.
(4)

Given that $y = 2$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = 2$ at $x = 0$

(b) find a series solution for y in ascending powers of x, up to and including the term in x^3.
(5)

Leave blank

Question 4 continued

Leave blank

5. (a) Find, in the form $y = \mathrm{f}(x)$, the general solution of the equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + 2y\tan x = \sin 2x, \qquad 0 < x < \frac{\pi}{2}$$

(6)

Given that $y = 2$ at $x = \frac{\pi}{3}$

(b) find the value of y at $x = \frac{\pi}{6}$, giving your answer in the form $a + k\ln b$, where a and b are integers and k is rational.

(4)

Leave blank

Question 5 continued

Leave blank

6. The complex number $z = e^{i\theta}$, where θ is real.

(a) Use de Moivre's theorem to show that

$$z^n + \frac{1}{z^n} = 2\cos n\theta$$

where n is a positive integer.

(2)

(b) Show that

$$\cos^5\theta = \frac{1}{16}(\cos 5\theta + 5\cos 3\theta + 10\cos\theta)$$

(5)

(c) Hence find all the solutions of

$$\cos 5\theta + 5\cos 3\theta + 12\cos\theta = 0$$

in the interval $0 \leqslant \theta < 2\pi$

(4)

Leave blank

Question 6 continued

Leave blank

7. (a) Find the value of λ for which $\lambda t^2 e^{3t}$ is a particular integral of the differential equation

$$\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 9y = 6e^{3t}, \qquad t \geqslant 0$$

(5)

(b) Hence find the general solution of this differential equation.

(3)

Given that when $t = 0$, $y = 5$ and $\dfrac{dy}{dt} = 4$

(c) find the particular solution of this differential equation, giving your solution in the form $y = f(t)$.

(5)

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Question 7 continued

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8.

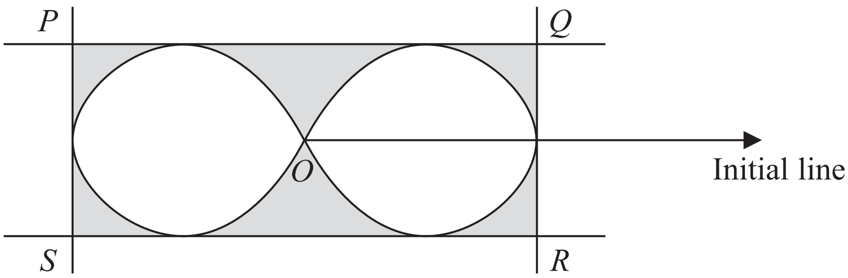

Figure 1

Figure 1 shows a closed curve C with equation

$$r = 3(\cos 2\theta)^{\frac{1}{2}}, \qquad \text{where } -\frac{\pi}{4} < \theta \leqslant \frac{\pi}{4}, \ \frac{3\pi}{4} < \theta \leqslant \frac{5\pi}{4}$$

The lines PQ, SR, PS and QR are tangents to C, where PQ and SR are parallel to the initial line and PS and QR are perpendicular to the initial line. The point O is the pole.

(a) Find the total area enclosed by the curve C, shown unshaded inside the rectangle in Figure 1.

(4)

(b) Find the total area of the region bounded by the curve C and the four tangents, shown shaded in Figure 1.

(9)

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Question 8 continued

Q8

(Total 13 marks)

TOTAL FOR PAPER: 75 MARKS

END

Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						6 6 6 8 / 0 1	Signature	

Paper Reference(s)

6668/01

Edexcel GCE

Further Pure Mathematics FP2

Advanced/Advanced Subsidiary

Friday 21 June 2013 – Morning

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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Turn over

PEARSON

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1. (a) Express $\dfrac{2}{(2r+1)(2r+3)}$ in partial fractions.

(2)

(b) Using your answer to (a), find, in terms of n,

$$\sum_{r=1}^{n} \frac{3}{(2r+1)(2r+3)}$$

Give your answer as a single fraction in its simplest form.

(3)

Leave blank

2. $$z = 5\sqrt{3} - 5\text{i}$$

Find

(a) $|z|$,

(1)

(b) $\arg(z)$, in terms of π.

(2)

$$w = 2\left(\cos\frac{\pi}{4} + \text{i}\sin\frac{\pi}{4}\right)$$

Find

(c) $\left|\frac{w}{z}\right|$,

(1)

(d) $\arg\left(\frac{w}{z}\right)$, in terms of π.

(2)

Leave blank

3.

$$\frac{d^2y}{dx^2} + 4y - \sin x = 0$$

Given that $y = \frac{1}{2}$ and $\frac{dy}{dx} = \frac{1}{8}$ at $x = 0$,

find a series expansion for y in terms of x, up to and including the term in x^3.

(5)

Leave blank

4. (a) Given that

$$z = r(\cos\theta + \mathrm{i}\sin\theta), \qquad r \in \mathbb{R}$$

prove, by induction, that $z^n = r^n(\cos n\theta + \mathrm{i}\sin n\theta), \qquad n \in \mathbb{Z}^+$

(5)

$$w = 3\left(\cos\frac{3\pi}{4} + \mathrm{i}\sin\frac{3\pi}{4}\right)$$

(b) Find the exact value of w^5, giving your answer in the form $a + \mathrm{i}b$, where $a, b \in \mathbb{R}$.

(2)

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Question 4 continued

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5. (a) Find the general solution of the differential equation

$$x\frac{dy}{dx} + 2y = 4x^2$$

(5)

(b) Find the particular solution for which $y = 5$ at $x = 1$, giving your answer in the form $y = f(x)$.

(2)

(c) (i) Find the exact values of the coordinates of the turning points of the curve with equation $y = f(x)$, making your method clear.

(ii) Sketch the curve with equation $y = f(x)$, showing the coordinates of the turning points.

(5)

Leave blank

Question 5 continued

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6. (a) Use algebra to find the exact solutions of the equation

$$|2x^2 + 6x - 5| = 5 - 2x$$

(6)

(b) On the same diagram, sketch the curve with equation $y = |2x^2 + 6x - 5|$ and the line with equation $y = 5 - 2x$, showing the x-coordinates of the points where the line crosses the curve.

(3)

(c) Find the set of values of x for which

$$|2x^2 + 6x - 5| > 5 - 2x$$

(3)

Leave blank

Question 6 continued

Leave blank

7. (a) Show that the transformation $y = xv$ transforms the equation

$$4x^2\frac{d^2y}{dx^2} - 8x\frac{dy}{dx} + (8 + 4x^2)y = x^4 \qquad \text{(I)}$$

into the equation

$$4\frac{d^2v}{dx^2} + 4v = x \qquad \text{(II)}$$

(6)

(b) Solve the differential equation (II) to find v as a function of x.

(6)

(c) Hence state the general solution of the differential equation (I).

(1)

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Question 7 continued

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8.

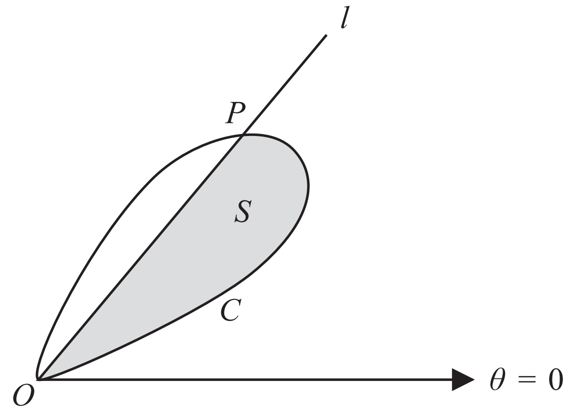

Figure 1

Figure 1 shows a curve C with polar equation $r = a\sin 2\theta,\ 0 \leqslant \theta \leqslant \frac{\pi}{2}$, and a half-line l.

The half-line l meets C at the pole O and at the point P. The tangent to C at P is parallel to the initial line. The polar coordinates of P are (R, ϕ).

(a) Show that $\cos\phi = \dfrac{1}{\sqrt{3}}$

(6)

(b) Find the exact value of R.

(2)

The region S, shown shaded in Figure 1, is bounded by C and l.

(c) Use calculus to show that the exact area of S is

$$\frac{1}{36}a^2\left(9\arccos\left(\frac{1}{\sqrt{3}}\right) + \sqrt{2}\right)$$

(7)

Leave blank

Question 8 continued

Q8

(Total 15 marks)

TOTAL FOR PAPER: 75 MARKS

END

Further Pure Mathematics FP2

Candidates sitting FP2 may also require those formulae listed under Further Pure Mathematics FP1 and Core Mathematics C1–C4.

Area of a sector

$$A = \frac{1}{2}\int r^2 \ \mathrm{d}\theta \qquad \text{(polar coordinates)}$$

Complex numbers

$$\mathrm{e}^{\mathrm{i}\theta} = \cos\theta + \mathrm{i}\sin\theta$$

$$\{r(\cos\theta + \mathrm{i}\sin\theta)\}^n = r^n(\cos n\theta + \mathrm{i}\sin n\theta)$$

The roots of $z^n = 1$ are given by $z = \mathrm{e}^{\frac{2\pi k \mathrm{i}}{n}}$, for $k = 0, 1, 2, \ldots, n-1$

Maclaurin's and Taylor's Series

$$\mathrm{f}(x) = \mathrm{f}(0) + x\,\mathrm{f}'(0) + \frac{x^2}{2!}\mathrm{f}''(0) + \ldots + \frac{x^r}{r!}\mathrm{f}^{(r)}(0) + \ldots$$

$$\mathrm{f}(x) = \mathrm{f}(a) + (x-a)\,\mathrm{f}'(a) + \frac{(x-a)^2}{2!}\mathrm{f}''(a) + \ldots + \frac{(x-a)^r}{r!}\mathrm{f}^{(r)}(a) + \ldots$$

$$\mathrm{f}(a+x) = \mathrm{f}(a) + x\,\mathrm{f}'(a) + \frac{x^2}{2!}\mathrm{f}''(a) + \ldots + \frac{x^r}{r!}\mathrm{f}^{(r)}(a) + \ldots$$

$$\mathrm{e}^x = \exp(x) = 1 + x + \frac{x^2}{2!} + \ldots + \frac{x^r}{r!} + \ldots \qquad \text{for all } x$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots + (-1)^{r+1}\frac{x^r}{r} + \ldots \qquad (-1 < x \leq 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots + (-1)^r \frac{x^{2r+1}}{(2r+1)!} + \ldots \qquad \text{for all } x$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots + (-1)^r \frac{x^{2r}}{(2r)!} + \ldots \qquad \text{for all } x$$

$$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \ldots + (-1)^r \frac{x^{2r+1}}{2r+1} + \ldots \qquad (-1 \leq x \leq 1)$$

Further Pure Mathematics FP1

Candidates sitting FP1 may also require those formulae listed under Core Mathematics C1 and C2.

Summations

$$\sum_{r=1}^{n} r^2 = \tfrac{1}{6}n(n+1)(2n+1)$$

$$\sum_{r=1}^{n} r^3 = \tfrac{1}{4}n^2(n+1)^2$$

Numerical solution of equations

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Conics

	Parabola	**Rectangular Hyperbola**
Standard Form	$y^2 = 4ax$	$xy = c^2$
Parametric Form	$(at^2, 2at)$	$\left(ct, \dfrac{c}{t}\right)$
Foci	$(a, 0)$	Not required
Directrices	$x = -a$	Not required

Matrix transformations

Anticlockwise rotation through θ about O: $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

Reflection in the line $y = (\tan\theta)x$: $\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$

In FP1, θ will be a multiple of 45°.

Core Mathematics C4

Candidates sitting C4 may also require those formulae listed under Core Mathematics C1, C2 and C3.

***Integration* (+ *constant*)**

$\mathbf{f}(x)$	$\int \mathbf{f}(x)\ \mathrm{d}x$
$\sec^2 kx$	$\frac{1}{k}\tan kx$
$\tan x$	$\ln\lvert\sec x\rvert$
$\cot x$	$\ln\lvert\sin x\rvert$
$\operatorname{cosec} x$	$-\ln\lvert\operatorname{cosec} x + \cot x\rvert, \quad \ln\lvert\tan(\tfrac{1}{2}x)\rvert$
$\sec x$	$\ln\lvert\sec x + \tan x\rvert, \quad \ln\lvert\tan(\tfrac{1}{2}x + \tfrac{1}{4}\pi)\rvert$

$$\int u\frac{\mathrm{d}v}{\mathrm{d}x}\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\mathrm{d}x$$

Core Mathematics C3

Candidates sitting C3 may also require those formulae listed under Core Mathematics C1 and C2.

Logarithms and exponentials

$$e^{x \ln a} = a^x$$

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad (A \pm B \neq (k + \tfrac{1}{2})\pi)$$

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

Differentiation

f(x)	**f′(x)**
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\text{cosec}^2 x$
$\text{cosec}\, x$	$-\text{cosec}\, x \cot x$
$\dfrac{f(x)}{g(x)}$	$\dfrac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$

Core Mathematics C2

Candidates sitting C2 may also require those formulae listed under Core Mathematics C1.

Cosine rule

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Binomial series

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N})$$

$$\text{where } \binom{n}{r} = {}^n\mathrm{C}_r = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1\times 2}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{1\times 2\times\ldots\times r}x^r + \ldots \qquad (|x| < 1, n \in \mathbb{R})$$

Logarithms and exponentials

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Geometric series

$$u_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

Numerical integration

The trapezium rule: $\int_a^b y \, \mathrm{d}x \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \frac{b-a}{n}$

Core Mathematics C1

Mensuration

Surface area of sphere = $4\pi r^2$

Area of curved surface of cone = $\pi r \times$ slant height

Arithmetic series

$u_n = a + (n - 1)d$

$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n[2a + (n - 1)d]$